Evolutionary competition in a mixed market with socially concerned firms

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Abstract
In this paper we study an oligopoly market where profit-maximizing firms and socially concerned firms compete in quantities. Confronting remarks by Milton Friedman and Gary Becker, we are using an evolutionary setting to investigate the endogenous choice of the proper objective of business firms and the influence of product differentiation on the long run survival of firms which pursue non-profit motives. We find that firms which consider a combination of profit and consumer welfare can indeed have larger market shares and profits than their profit-maximizing rivals. One insight is that it might pay off for shareholders to consider stakeholder welfare, but that the level of social concern should not be too high. Based on a strategy’s profitability, we consider asynchronous evolutionary updating with firms selecting Nash quantities or choosing best replies to the expected market quantity. Here we observe that the consumers’ willingness to pay a price premium for products is crucial for the long run survival of socially concerned firms. Depending on the degrees of product differentiation and social concern, long run outcomes consist either of both types of firms or only one type of firm. If the firms’ propensity to switch between a social or a profit-maximizing strategy is sufficiently large, steady states are unstable and even complicated dynamics can occur.

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1. Introduction

Corporate Social Responsibility (henceforth CSR) plays an increasingly important role for firms in today’s globalized market environment. Investors and consumers alike pressure companies to consider social and environmental issues. Documenting the importance of this trend, the consulting firm KPMG reports that nearly 95% of the largest 250 companies worldwide issued CSR reports, up from 80% in 2008 and 50% in 2005 (KPMG, 2011, 2008, 2005). Hence, CSR has become mainstream in the corporate world.

Empirical and theoretical work in economics, industrial organization, and management research is still divided about the question if pursuing a social strategy can serve shareholder value. Milton Friedman’s remark that “the social responsibility of business is to increase its profits” succinctly summarizes the arguments of one group of researchers (Friedman, 1970).
They contend that first-mover advantages of CSR activities quickly erode over time and make such investments unattractive (see, e.g., Kopel, 2011; The Economist, 2008). Other scholars emphasize the business case of CSR and present it as a strategic weapon to achieve a competitive advantage and increase profits (e.g. Husted and Allen, 2011). Empirical studies investigating the link between Corporate Social Performance and Corporate Financial Performance do not show clear-cut results (McWilliams et al., 2006; McWilliams and Siegel, 2000), although newer studies show a tendency towards a positive relation under certain conditions (e.g. Servaes and Tamayo, 2013; Brammer and Millington, 2008). Theoretical work starts to realize the long-term investment characteristic of CSR activities, but primarily uses static models with only a few exceptions (e.g. Wirl et al., 2013). The goal of our paper is to gain insights into the drivers for the recent trend in corporate social responsibility and to shed some light on the potential dynamics of an industry involving firms with profit and non-profit motives. In particular, we want to evaluate Gary Becker’s claim that “Companies that combine the profit motive with environmental and other concerns can thrive in a competitive environment only if they are able to attract […] customers that also value these other corporate goals.” (Becker, 2008; see also Frank, 2004). Hence, the research questions we want to answer in this paper are: “What are the possible long run outcomes in a mixed industry where firms pursue non-profit motives like consumer welfare?”, “Would profit-maximizing owners want their firm to be run in a socially responsible way?”, “What is the influence of the products’ degree of differentiation and the firms’ level of social concern?”. To elaborate, we consider an evolutionary setting based on a mixed N-firm oligopoly model, where a group of socially concerned firms (S-firms) competes with a group of profit-maximizing firms (P-firms). In contrast to a P-firm, a S-firm maximizes its profit plus a fraction of consumer surplus (e.g. Königstein and Müller, 2001; Kopel and Brand, 2012 and the literature cited there). To account for the fact that consumers might prefer a product of a S-firm, we assume that the products offered by the two groups are horizontally and vertically differentiated. First, we characterize the Cournot-Nash outcome in the general N-firms oligopoly. As this static mixed setting has not been considered in the literature, we think that these results are interesting on their own. For the evolutionary model, we focus on the duopoly case. We discuss the equilibria arising in a two-stage model, where in the first stage of the game the owners choose simultaneously whether their firm is run as a S-firm or as a P-firm. In the second stage they choose the quantity to produce in order to maximize the firm’s expected payoff. Here, the share of consumer surplus included in the S-firm’s objective function as well as the degree of product differentiation are assumed to be exogenously given. Depending on the combination of these parameters, two cases can arise. The game is either dominance solvable, with both firms selecting to be a P-firm or both selecting to be a S-firm. The game can also be of anti-coordination type, with a mixed-strategy Nash equilibrium in which both strategies, P-firm and S-firm, are played with positive probability. This demonstrates that depending on the product characteristics and the level of social concern the industry evolves either towards a homogeneous population of firms or towards a heterogeneous state where both type of firms coexist in the industry. The main insight here is that an increase in the willingness to pay for products of S-firms makes it more likely that all firms adopt a social strategy. Additionally, such a strategy might even be more profitable than pursuing a profit-maximizing strategy.

Following Droste et al. (2002), we then address the issue of robustness of CSR behavior under evolutionary pressure in the mixed duopoly market. Differently from their paper, in our setting firms have in common the same information and are endowed with the same degree of rationality, but they can select different objective functions. An alternative interpretation is that the owners pick a manager with a preference for pursuing different objectives. Firms are randomly matched to play the mixed Cournot duopoly game with differentiated products. In line with the ‘indirect evolutionary approach’ (e.g. Bester and Güth, 1998; Königstein and Müller, 2000, or more recently Alger and Weibull, 2013), the fraction of P-firms and S-firms evolves over time according to the realized profits in the quantity game through an asynchronous updating mechanism (see Hommes, 2009). According to Königstein and Müller (2000), the indirect evolutionary approach studies “...evolution of behavior when the motives (preferences) that drive individual decisions differ from the forces that determine long-run survival of motives (preferences) within a society.” (p. 238). This approach is useful to analyze the evolutionary dynamics of behaviors (rather than strategies), where, in general, evolution is determined by ‘objective’ utility functions whereas players’ behavior depends on their ‘subjective’ utility. In this vein, S-firms choose their quantities to maximize a subjective composite objective function (of e.g. management) that includes consumer surplus, but the evolutionary success of each strategy (S and P) is determined by realized profits and hence an objective ‘monetary’ measure of fitness (of e.g. investors).

With respect to firms’ behavioral rules, we consider two situations to check for the robustness of our results. First, we briefly study the dynamics of the evolutionary game under the assumption of competition between “Nash” players. When firms are “Nash” players, they are assumed to learn the type of the player they are matched with and select the Cournot-Nash outcome prescribed by the chosen strategy (P-firm or S-firm) at the quantity stage. In this case, the dynamic model assumes the form of an unidimensional map for the fraction of S-firms. We show that when the game is dominance solvable, the pure strategy Nash equilibria are always stable attractors, regardless of firms’ intensity of choice. In the case of an anti-coordination game, the mixed Nash equilibrium is stable if the firms’ propensity to switch strategies is sufficiently small. This equilibrium loses stability through a flip bifurcation for sufficiently high intensity of choice. Second, we analyze the dynamics of the evolutionary game under the assumption of “best reply” players (see Droste et al., 2002; Hommes et al., 2011). In this case players do not know the type of their opponent and best-reply to an expected (or average) population quantity. Here both quantity dynamics and evolutionary dynamics are taken into account. A stability analysis of the equilibria is provided as well as a comparison between the dynamics with Nash players and best reply players. Also in the case of best reply players there exists a specific threshold level for the intensity of choice after which the mixed Nash equilibrium loses stability. Confirming the robustness of the result, it again turns out the consumers’ willingness to pay a
higher price for the products of S-firms and the level of social concern crucially influence the firms' incentive to implement a social strategy.

2. Related literature

Our paper is related to the literature on mixed oligopoly markets. The majority of research has used static models to study the impact of firms that pursue non-profit motives on the market structure, the firm conduct, performance, and welfare. For example, the classic papers by Matsumura (1998) and Cremer et al. (1991) study competition between private (profit-maximizing) firms and (partially privatized) public firms that consider welfare. See e.g. De Fraja (2009), Saha (2009), and Kalashnikov et al. (2009) for further references. Riechmann (2006) analyzes a Cournot duopoly model including a firm that maximizes relative payoffs. Relatedly, Zhang and Zhang (2006) introduce a model of strategic alliances between firms where each alliance member maximizes its own profit and a share of its partner's profit. Our paper differs as we study an oligopoly with private firms and socially concerned firms where the latter are hybrid organizations somewhere between private and public firms. Moreover, we are concerned with endogenous selection of the organizational form in terms of the firm's objective\(^1\) in an evolutionary setting. The papers by Goering (2008a,b), Kopel and Brand (2012), Lien (2002), and Brekke et al. (2012) assume similar objective functions for their firms with non-profit motives as we do, but consider static Cournot models where the objective of firms is fixed. Three recent papers, Chirco et al. (2013), Kopel and Szidarovsky (2013), and Lambertini and Tampieri (2011) consider endogenous heterogeneity in objectives of firms, but their setups are static in nature since they study deviation-proof equilibria based on the concept of cartel stability. The literature on dynamic mixed oligopoly is quite limited. For example, Casadesus-Masanell and Ghemawat (2006) analyze a dynamic mixed duopoly game where one competitor prices at marginal costs. In contrast to our setting, they consider a dynamic optimization framework and objectives are fixed.\(^2\)

More related to our paper are the contributions dealing with evolutionary selection and behavioral heterogeneity. For example, Schaffer (1989) demonstrates that if firms have market power, then profit-maximizing firms are not necessarily the best survivors.\(^3\) In the same vain, Heifetz et al. (2007) argue that in almost every strategic interaction, payoff maximization cannot be justified on evolutionary grounds. They show that under any payoff-monotone selection dynamics, the population will not converge to payoff maximizing behavior. Rhode and Stegeman (2001) study a differentiated duopoly and show that if firms' choices follow a Darwinian process, then the long run outcome is not a Nash equilibrium and the evolution of objectives distorts behavior towards revenue maximization. Relatedly, Königstein and Müller (2001) adopt an indirect evolutionary approach to demonstrate that it is beneficial for firms to not only maximize profit but to include a share of consumer welfare as well. In contrast to their work, in our model the share of consumer welfare included in a firm's objective is fixed. Moreover, we consider the dynamics of the evolutionary game with endogenous switching of firm types whereas they solely focus on evolutionary stability. Droste et al. (2002) investigate an evolutionary model of Cournot competition where in every period a large population is matched in pairs. For a simple setting with two firms which can select between two behavioral rules to specify the current period's quantity – a best reply rule and a (costly, since sophisticated) Nash rule – the authors show that complicated and endogenous fluctuations may arise. Bischli et al. (2003a,b) investigate the global dynamics of a two-population model with intra-group and inter-group spillovers and demonstrate too that complicated dynamics may occur. Hommes et al. (2011) study an evolutionary model of Cournot competition where based on past performance firms switch between different expectation rules concerning aggregate output of its rivals. They find that Theoecharis' result on the instability of the Nash equilibrium for more than three firms persists under evolutionary competition between heterogeneous (costly and costless) expectation heuristics. In our setup firms do not choose among (costly) behavioral or expectation rules, but firms choose their organizational structure represented by an objective function. Based on the resulting profits, owners either choose a social strategy including also consumer welfare (and hence another stakeholder group) or select a profit motive simply serving shareholders. We derive the conditions such that in an evolutionary setting socially concerned firms may dominate in the industry or may coexist with profit-maximizing firms. Finally, our paper is also related to Fanelli (2010) who studies the influence of consumers' preferences on the incentives of (profit-maximizing) firms to offer a more expensive, but higher-valued product. Using a (simple, synchronous) replicator dynamics, he only finds two types of equilibria: (i) both type of firms coexist or (ii) only standard profit-maximizing firms are in the market.

3. Static mixed oligopoly

Assume that there are \(N \geq 2\) firms, which are divided into two groups of size \(n_p = m \geq 0\) and \(n_s = N - m \geq 0\). The firms in group \(P\) are standard profit maximizing firms (P-firms). The firms in group \(S\) are socially concerned firms (S-firms) with the objective to maximize profit plus a share of consumer surplus. The products of firms within each group \(j (j = P, S)\) are...
assumed to be homogenous, whereas the products of P-firms and S-firms are (horizontally and vertically) differentiated. The utility of a consumer who chooses a bundle of products from both groups is

\[ U(x_{P1}, \ldots, x_{Pm}, x_{S1}, \ldots, x_{SN-m}) = \sum_{k=1}^{m} x_{P_k} + a \sum_{k=1}^{N-m} x_{S_k} - \frac{1}{2} \sum_{k=1}^{m} x_{P_k}^2 + \sum_{k=1}^{N-m} x_{S_k}^2 - \sum_{i \neq j} x_{P_i} x_{S_j} - \gamma \sum_{i=1}^{m} x_{P_i} x_{S_j} \]

under the budget constraint \( p_P \sum_{k=1}^{m} x_{P_k} + p_S \sum_{k=1}^{N-m} x_{S_k} \leq I \), where \( I \) is income and \( p_P \) and \( p_S \) are the prices of the products.

Taking into account that \( \partial U / \partial x_{Pj} = p_P \) and \( \partial U / \partial x_{Sj} = p_S \) for all \( j \), we obtain the inverse demands as (see similarly Häcker, 2000)

\[ p_P = 1 - x_P - \gamma x_S \]
\[ p_S = a - x_S - \gamma x_P, \quad a \geq 1, \quad \gamma \]

where \( x_P = \sum_{k=1}^{m} x_{P_k} \) is the total quantity of the P-firms, \( x_S = \sum_{k=1}^{N-m} x_{S_k} \) is the total quantity of the S-firms, and \( x_i \) is the quantity produced by the \( i \)-th firm in group \( l = P, S \). The parameter \( a \geq 1 \) is the maximum price consumers are willing to pay for products of an S-firm. It measures the degree of vertical differentiation or the quality of the product offered by S-firms. Since \( a \geq 1 \), in line with our research questions we allow for the fact that consumers perceive the quality of products offered by S-firms as higher than the quality of products supplied by P-firms. The parameter \( \gamma \in [-1, 1] \) measures the degree of horizontal differentiation between products of the two groups. Note that for \( \gamma \in [-1, 0) \) the products are complements and for \( \gamma \in (0, 1] \) the products are substitutes. The groups act in independent markets for \( \gamma = 0 \). We assume that firms within a group have the same linear costs,

\[ C_l(x_l) = c_l x_l, \quad l = P, S, \]

with \( c_P < 1 \) and \( c_S < a \). The objective functions of firms are as follows. A P-firm maximizes its profit, i.e. a firm \( k \) in group \( P \) maximizes

\[ \Pi_{P_k} = (p_P - c_P)x_{P_k}, \quad k = 1, \ldots, m. \]

A firm in group \( S \) maximizes the sum of its profit plus a share \( \theta_i \in [0, 1] \) of the consumer surplus \( CS \), i.e.

\[ V_{S_i} = \Pi_{S_i} + \theta_i \cdot CS = (p_S - c_S) x_{S_i} + \theta_i \cdot CS \]

where the consumer surplus can be derived as

\[ CS = U(x_{P1}, \ldots, x_{Pm}, x_{S1}, \ldots, x_{SN-m}) - p_P \sum_{k=1}^{m} x_{P_k} - p_S \sum_{k=1}^{N-m} x_{S_k} - \sum_{i \neq j} x_{P_i} x_{S_j} - \gamma \sum_{i=1}^{m} x_{P_i} x_{S_j} = \frac{1}{2} \left( \sum_{k=1}^{m} x_{P_k}^2 + 2\gamma \sum_{k=1}^{m} x_{P_k} \sum_{i=1}^{N-m} x_{S_i} + \left( \sum_{k=1}^{N-m} x_{S_k} \right)^2 \right). \]

With this type of objective function we try to capture a firm type referred to as syncretic stewardship model by Berger et al. (2007), referring to an organization which embraces economic as well as non-economic goals. Likewise, we can think of a hybrid organizational structure, which enables the firm to pursue profit and non-profit motives (Bromberger, 2011; Porter and Kramer, 2011). A (linear) combination of profit and non-profit goals has been used frequently in the (recent) literature to capture a firm’s broader objective, see e.g. Kopel and Brand (2012), Brekke et al. (2012), Saha (2014), Goering (2007, 2008a,b, 2012), Lambertini and Tampieri (2010), Kelsey and Milne (2008), Lien (2002). Königstein and Müller (2001) use an evolutionary setting to show that firms can benefit from considering profit plus consumer surplus; see also Heifetz et al. (2007). Finally, Lambertini (2013) considers a mixed duopoly and studies the interplay between a negative environmental externality of production and firms’ socially responsible behavior (see, in particular, Chapter 5). For tractability, we will assume that the level of social concern is the same for all S-firms, i.e. \( \theta_i = \theta \in [0, 1] \) for all \( i = 1, \ldots, N-m \).

The total and individual outputs of P-firms and S-firms can now be easily derived. From the first order condition \( \partial \Pi_{P_k} / \partial x_{P_k} = 0 \) for a P-firm we obtain the reaction function

\[ x_{P_k} = \frac{1-c_P}{2} - \frac{1}{2} a x_{P_{-k}} - \frac{\gamma}{2} x_S \]

with \( x_{P_{-k}} = \sum_{j \neq k} x_{P_j} \), which can be written as

\[ x_{P_k} = 1-c_P - x_P - \gamma x_S = p_P - c_P. \]

Summing over \( k = 1, \ldots, m \) yields

\[ (1+m)x_P + \gamma m x_S = m(1-c_P). \]

From the first order condition \( \partial V_{S_i} / \partial x_{S_i} = 0 \) for an S-firm, we obtain the reaction function

\[ x_{S_i} = \frac{a-c_S}{2-\theta} - \frac{1-\theta}{2} x_{S_{-i}} - \frac{\gamma (1-\theta)}{2-\theta} x_P \]
S-firm can also be derived easily since we know from the first order condition that $p_S - c_S = \theta(X_S + \gamma X_P)$.

The equilibrium profits of a profit-maximizing firm and a socially concerned firm are costs

Proposition 1. In the proposed mixed N-firm oligopoly with m profit-maximizing firms and $(N - m)$ socially concerned firms the equilibrium quantities are given by

$$x_{Sp} = \frac{X_p}{m} = \frac{(1 - c_p)(1 + (N - m)(1 - \theta)) - (a - c_S)(N - m)\gamma}{(m + 1)(1 + (N - m)(1 - \theta)) - (N - m)(1 - \theta)\gamma^2}$$

$$x_{Si} = \frac{X_S}{N - m} = \frac{(a - c_S)(m + 1) - (1 - c_p)(1 - \theta)\gamma}{(m + 1)(1 + (N - m)(1 - \theta)) - (N - m)(1 - \theta)\gamma^2}$$

The equilibrium profits of a profit-maximizing firm and a socially concerned firm are, respectively,

$$\pi_{Sp} = x_{Sp}^2 C_p$$

$$\pi_{Si} = [(1 - \theta(N - m)x_{Si} - \theta \gamma mx_{P})] \cdot x_{Si}$$

It is easy to see that (partially) pursuing the welfare of consumers in addition to profits yields a strategic benefit in a quantity-setting oligopoly (e.g. Kopel and Brand, 2012; Lambertini, 2013). Moreover, given that pursuing an extended objective function like $V_{Si}$ in (5) can alternatively be interpreted as having a manager with consumer-regarding preferences, the connection to the strategic incentives literature becomes obvious (e.g. Fershtman and Judd, 1987; Heifetz et al, 2007). Consider, for example, the simple case $a = 1$ (no vertical differentiation), $\gamma = 1$ (homogenous products), and identical unit costs $c_p = c_S$. Then it can be shown that $x_{Si} > 0$ for all $\theta \in (0, 1]$, but $x_{Sp} > 0$ only if the level of social concern is sufficiently...
small, $\theta < 1/(N-m)$. If the level of social concern is sufficiently high, then the profit-maximizing firm exits the market. Comparing the firms’ equilibrium quantities given in the proposition we obtain $x_{pk} < x_{Si}$ if

$$\theta > \frac{(1-c_p)(N-m+1+m\gamma)-(a-c_S)(m+1+(N-m)\gamma)}{(1-c_p)(N-m+m\gamma)},$$

which is always guaranteed if $a=1$, $\gamma=1$, and $c_p = c_S$. Since in this simple homogenous-products case there is a unique market price, a higher market quantity results in a higher profit. Hence, being socially concerned is more profitable than just focusing on profits alone.

To characterize the feasibility regions of quantities and profits in more detail, we take a closer look at the parameter space of unit costs $c_p$ and $c_S$. Note that it can be argued that socially concerned firms have higher or lower costs than a profit-maximizing firm. Socially concerned firms may have higher costs, e.g., due to a lack of focus on a single objective or the potential for managerial entrenchment (e.g. Tirole, 2001). On the other hand, socially concerned firms are able to attract and retain highly motivated employees and, as a result, may have lower costs (e.g., Berger et al., 2007). Therefore, in the analysis below we study the general case. First, notice that $x_{pk} \geq 0$ iff $(1-c_p)(1+(N-m)(1-\theta)) \geq (a-c_S)(N-m)\gamma$. This yields

$$c_p \leq \frac{1+(N-m)[1-\theta-a\gamma]}{1+(N-m)(1-\theta)} + \frac{\gamma(N-m)}{1+(N-m)(1-\theta)} c_S. \tag{12}$$

Also $\pi_{pk} \geq 0$ in this case. Second, $x_{Si} \geq 0$ iff

$$c_p \geq \frac{\gamma m(1-\theta)-a(1+m)}{(1-\theta)m} + \frac{1+m}{(1-\theta)m} c_S. \tag{13}$$

Third, $\pi_{Si} \geq 0$ if (13) holds and additionally

$$(1-\theta(N-m))x_{Si} - \theta my_{pk} \geq 0. \tag{14}$$

Notice that it might happen that $x_{Si} \geq 0$, but $\pi_{Si} < 0$ (see, e.g., Kopel and Brand, 2012).

In Fig. 1 we depict the parameter space of unit costs for a representative set of parameter values (e.g. $a=1, N=2$, $m=1, \gamma=1, \theta=0.3$) and show three lines, $x_P = 0$, $x_S = 0$, and $\pi_S = 0$, associated with the inequalities (12), (13), and (14) respectively. All lines intersect in the point $I$. If the unit costs of the P-firms (S-firms) become too high, then the S-firms (P-firms) take over the market; see the regions above the line $x_P = 0$ (to the right of line $x_S = 0$). In the kite-shaped region of the figure, the combinations of unit costs lead to positive quantities. Note however that the profit of a socially firm might still become negative as indicated by the line $\pi_S = 0$ representing (14). Finally, we also depict the curve of identical profits, $\pi_P = \pi_S$, for $a=1, N=2, m=1, \gamma=1, \theta=0.3$. Observe that for identical unit costs ($c_P = c_S$) represented by the diagonal in Fig. 1 the socially concerned firm obtains a higher profit.

Using (12)–(14), the points $c_P^1, c_P^2, c_S^2$ on the axes can be derived easily as

$$c_P^1 = 1 \frac{(N-m)a\gamma}{1+(N-m)(1-\theta)}, \quad c_P^1 = a \frac{(1-\theta)m\gamma}{1+m}, \quad c_S^2 = a \frac{m\gamma}{1+m-(1+1-\gamma)m(N-m)\theta}.$$

These points can be used to gain insights into the influence of parameters on the set of feasible equilibria. For example, in line with intuition, as the quality difference measured by $a$ increases, the socially concerned firm becomes a stronger competitor (since the lines rotate and shift to the right through point $I$). Likewise, in terms of market quantity, increases in the level of social concern weaken P-firms ($c_P^1$ moves downwards), but make S-firms stronger ($c_P^1$ moves right). In terms of profits, higher $\theta$ increase the likelihood of negative profits for the socially concerned firm ($c_S^2$ moves left). On the other hand, for example for $a=1, N=2, m=1, \gamma=1$ the intersection point $c_S^2 = \theta(1-\theta)/3-2\theta > 0$. This shows that if S-firms have nearly identical unit costs as P-firms, they achieve higher profits due to the commitment effect of partially pursuing consumer welfare for all $\theta \in (0, 1)$. It can be shown that in a duopoly situation, by choosing $\theta = 1/3$ the S-firm can even achieve the Stackelberg outcome (see Lambertini, 2013; Kopel and Brand, 2012).

4. Evolutionary mixed duopoly

In this section, we endogenize the choice of the objective function by the firms. More precisely, we assume that in the first stage of the game, the (profit-maximizing) owners or the board of directors either select a profit-maximizing strategy (P) or a social strategy including consumer welfare (S). An alternative interpretation is that the owners pick a manager with a preference for maximizing P or S (see, e.g., Schaffer, 1989). In the second stage, firms produce quantities maximizing the objective function associated with the chosen strategic orientation. We focus on an industry with two firms ($N=2$) using the same production technology ($c_P = c_S = c < 1$). It should be noted, however, that in our linear model the assumption of identical unit costs is no restriction of generality if products are vertically differentiated, $a > 1$.

The following scenarios can occur. First, if both firms are profit-maximizing P-firms we have $N = n_P = m = 2, n_S = 0$ and the market is described by a standard (homogenous) Cournot duopoly with an inverse demand given by $p_P = 1-x_{p1}-x_{p2}$. 

\[ \text{...} \]
The symmetric Cournot-Nash equilibrium is

\[ x_{pp} = x_{p1} = x_{p2} = \frac{1-c}{3} > 0 \]  

(15)

and profits are \( \pi_{pp} = ((1-c)/3)^2 \).

Second, if the owners of both firms want their firms to be run as socially concerned entities we have \( N = n_S = 2, n_P = m = 0 \), the products are again homogenous, and inverse demand is \( p_S = a - x_{S1} - x_{S2} \) with \( a \geq 1 \). Firms maximize (5) with \( \theta_1 = \theta_2 = \theta > 0 \). From the general expressions of the individual equilibrium quantities in (11), we obtain

\[ x_{SS} = x_{S1} = x_{S2} = \frac{a-c}{3-2\theta} > 0 \]  

(16)

and the equilibrium profit

\[ \pi_{SS} = \frac{(a-c)^2(1-2\theta)}{(3-2\theta)^2}. \]

Note that if the level of social concern is too high (\( \theta > 1/2 \)), then the profit becomes negative. Finally, if the owners of one firm select profit maximization and the owners of the other firm select a social strategy, then \( n_P = m = 1, n_S = 1 \) and we obtain a mixed duopoly with inverse demands given by \( p_P = 1 - x_{p1} - \gamma x_{S1} \) and \( p_S = a - x_{S1} - \gamma x_{P1} \), i.e., products are horizontally and vertically differentiated. Using (11), the (asymmetric) Nash equilibrium is either

\[ (x_{PS}, x_{SP}) = (x_{P1}, x_{S1}) = \left( \frac{(1-c)(2-\theta) - \gamma(a-c)}{4 - \gamma^2(1-\theta)(1-2\theta)}, \frac{2(a-c) - (1-\theta)(1-c)\gamma}{4 - \gamma^2(1-\theta)(1-2\theta)} \right) \]  

(17)

or \( (x_{SP}, x_{PS}) = (x_{S1}, x_{P1}) \). It is easy to verify that the equilibrium quantities are both strictly positive whenever the following condition holds:

\[ 0 \leq c < \frac{2-a\gamma - \theta}{2-\gamma-\theta}. \]  

(18)

Moreover, at the equilibria \( (x_{PS}, x_{SP}) \) and \( (x_{SP}, x_{PS}) \), for any \( \gamma \in (-1, 1) \) the S-firm produces more than the P-firm, \( x_{SP} > x_{PS} \) (if \( \gamma = -1 \), this inequality holds provided that \( a > 1 \)). The corresponding profits of the firms are

\[ \pi_{PS} = \pi_{P1} = \frac{((1-c)(2-\theta) - \gamma(a-c))^2}{4 - \gamma^2(1-\theta)(1-2\theta)} \]

\[ \pi_{SP} = \pi_{S1} = \frac{((a-c)(2-\theta(2-\gamma^2)) - \gamma(1-c)[2(a-c) - \gamma(1-\theta)(1-c)]}{(4 - \gamma^2(1-\theta)(1-2\theta))^2} \]

Summarizing, we obtain the following payoff matrix, where \( \pi_{XY} \) denotes the profit realized by a firm playing strategy \( X \) when matched with a firm employing strategy \( Y \), with \( X, Y \in \{P, S\} \).

\[
\begin{array}{c|cc}
 & \text{S} & \text{P} \\
\hline
\text{S} & \pi_{SS}, \pi_{SS} & \pi_{SP}, \pi_{SP} \\
\text{P} & \pi_{PS}, \pi_{SP} & \pi_{PP}, \pi_{PP} \\
\end{array}
\]  

(19)

In the following subsection, we will show that the quantity dynamics always converges to a Nash equilibrium if each firm sticks to its selected strategy. We then introduce the possibility for firms to revise the decision of the first stage of the game. The dynamics of the selection of the strategic orientation (P or S) is modelled via a single-population evolutionary game, where in line with the indirect evolutionary approach the owners of the firms compare their current profit with the expected profit of selecting the other available strategic orientation. We begin by considering the case in which firms select the strategy to pursue (i.e. whether to be a P-firm or a S-firm) and choose the corresponding Nash equilibrium quantity in the second stage (hence we refer to the firms as ‘Nash’ players). First, we classify the resulting evolutionary game and characterize the evolutionary stable strategies (ESS). Second, we study the evolutionary dynamics of firms switching between the two strategies under the assumption of ‘Nash’ players. We then turn to the evolutionary dynamics with best reply players and study this case in detail.

We would like to mention that the assumption of a single population of firms is made for reasons of tractability. However, note that this rules out that the asymmetric equilibria of (19) can be the outcome of the evolutionary game like, for example, in a two-population setting (where in the long run one population solely consists of S-firms while the other population consists of P-firms). On the other hand, in our simpler model polymorphic states where only a share of firms selects the P-strategy (S-strategy) are still possible.

---

4 The weight \( \theta \) given to consumer surplus in a firm’s objective function might itself be a decision variable of the owners (e.g. Kopel and Brand, 2012; Königstein and Müller, 2001). We do not further pursue this idea in this paper.
4.1. Quantity dynamics

Consider the duopoly game where the strategy played by each firm (P or S) is fixed and the game is repeatedly played à la Cournot. We assume that firms have naive expectations about the other firms’ quantities meaning that each firm uses the current period’s realized quantities to assess how much competitors will produce in the next period. Then it turns out that quantity dynamics always converges to the unique Nash equilibrium.

**Proposition 2.** If the mixed duopoly game is played by firms updating the produced quantities according to the best reply dynamics with naive expectations, then the Nash equilibria \((x_{PP}, x_{SP}), (x_{PS}, x_{SP}), (x_{SP}, x_{PS}), (x_{SS}, x_{SS})\) are globally asymptotically stable.

**Proof.** See the Appendix.

4.2. Classification of the game and stable strategy profiles

Following Droste et al. (2002), we assume a single, infinite population of firms which are randomly matched in pairs at each time period to play the mixed duopoly game with product differentiation. Let \(r(t)\) denote the fraction of P-firms at time \(t\) and \(1 - r(t)\) the fraction of S-firms. The expected profit of a firm depends on the quantities played by both firms, which in the case of Nash players correspond to the Nash equilibrium quantities, and on the probability to be matched at time \(t\) with a firm playing strategy P or S. Hence, the expected profit of playing strategy \(P\), denoted by \(\pi_P\), is

\[
\pi_P = r\pi_{PP} + (1 - r)\pi_{PS} \\
= r \left(\frac{1 - c}{3}\right)^2 + (1 - r) \left(\frac{(1 - c)(2 - \theta) - \gamma(a - c)}{4 - \gamma^2(1 - \theta) - 2\theta}\right)^2.
\]  

The expected profit of playing strategy \(S\), denoted by \(\pi_S\), is

\[
\pi_S = \pi_{SP} + (1 - r)\pi_{SS} \\
= \left[\frac{(a - c)(2 - \theta)(2 - \gamma^2) - \gamma(1 - c)(1 - \theta)(1 - \theta)}{4 - \gamma^2(1 - \theta) - 2\theta}\right] \\
+ (1 - r) \frac{(a - c)^2(1 - 2\theta)}{(3 - 2\theta)^2}.
\]  

Any interior equilibrium \(r^*\) with coexistence of both types, P-firms and S-firms, is characterized by the condition \(\pi_P = \pi_S\). Solving yields

\[
r^* = \frac{\pi_{SS} - \pi_{PS}}{\pi_{PP} - \pi_{SP} + \pi_{SS} - \pi_{PS}}.
\]
Depending on the quality parameter $a$, the marginal cost $c$, the level of social concern $\theta$ and the degree of differentiation $\gamma$, it is possible to classify the game into the following categories and characterize the Evolutionary Stable Strategies (ESSs) in the various cases (see Weibull, 1995 for details and for the general proof). In particular, the game at hand with payoff matrix (19) can have three distinct forms.

- **Dominance solvable game I** – If $\pi_{SS} > \pi_{PS}$ and $\pi_{SP} > \pi_{PP}$ then $S$ dominates $P$. So $(S,S)$ is the only subgame-perfect Nash equilibrium and ESS. In this case the game can be a prisoner’s dilemma if, in addition to the conditions stated above, we also have $\pi_{PP} > \pi_{SS}$.

- **Dominance solvable game II** – If $\pi_{PS} > \pi_{SS}$ and $\pi_{PP} > \pi_{SP}$ then $P$ dominates $S$. So $(P,P)$ is the only subgame-perfect Nash equilibrium and ESS. Differently from the previous case, however, the model is never a prisoner’s dilemma, since it can be shown that the inequality $\pi_{PP} < \pi_{SS}$ is never satisfied.

- **Hawk-Dove game** – If $\pi_{PS} > \pi_{SS}$ and $\pi_{SP} > \pi_{PP}$ then $(P,S)$ and $(S,P)$ are asymmetric Nash equilibria; the only evolutionary stable state for the population is the equilibrium (22).

Within the proposed setup and under the various constraints on the parameters, it is possible to show that the inequalities $\pi_{SS} > \pi_{PS}$ and $\pi_{PP} > \pi_{SP}$ cannot be satisfied at the same time. This rules out a coordination game with two symmetric Nash equilibria of the form $(P,P)$ and $(S,S)$ and a mixed-strategy Nash equilibrium where $P$ is played with probability (22).

Fig. 2 depicts the occurrence of the three categories of games for the level of social concern $\theta \in [0, 1]$ and the degree of differentiation $\gamma \in [1, 1]$ with fixed values for the quality parameter $a$ and marginal costs $c$. In the gray region, $S$ dominates $P$ and all firms are socially concerned. In the black region, $P$ dominates $S$ and all firms are profit-maximizers. In the blue region, we obtain a Hawk-Dove game and we observe coexistence of $P$-firms and $S$-firms. Consider first the left panel of Fig. 2, where we assume no vertical differentiation ($a = 1$) and marginal cost $c = 0$. Even in this case, a social strategy survives in equilibrium. If products are complements ($\gamma < 0$), a mixed equilibrium with coexistence of $P$-firms and $S$-firms dominates. The intuition here is that quantity competition with complements is akin to price competition with substitutes (e.g. Singh and Vives, 1984) and the firm which chooses a social strategy takes the role of the price leader. In consequence, this results in higher profits for both firms compared to an equilibrium with identical strategy choices (see similarly Kopel and Brand, 2012; Lambertini, 2013). If products are substitutes ($0 < \gamma < 1$), all three equilibrium outcomes can occur. Interestingly, an equilibrium where all firms are socially concerned is possible only if the degree of competition measured by degree of substitution $\gamma$ is high, but the level of social concern is not too high. In this situation where $S$ is dominant, we also have $\pi_{PP} > \pi_{SS}$ so that the game is of a prisoner’s dilemma type. In other words, even though $S$ dominates $P$, individual profits are higher when all firms are profit maximizers (see similarly Kannaiainen and Pietarila, 2006). Finally, if the degree of social concern $\theta$ is too high, it is more profitable for firms to switch to a profit-maximizing strategy. The right panel of Fig. 2 shows a situation where products are vertically differentiated ($a = 1.3$) and marginal costs are positive ($c = 0.5$). Two observations are worth pointing out. First, the region in the parameters space where $S$ dominates $P$ is now enlarged and can be divided into two sub-regions. In the light gray region the game has a prisoner’s dilemma structure as before. In the dark gray region the game is a Hawk-Dove game and we observe coexistence of $P$-firms and $S$-firms. Consider first the left panel of Fig. 2, where we assume no vertical differentiation ($a = 1$) and marginal cost $c = 0$. Even in this case, a social strategy survives in equilibrium. If products are complements ($\gamma < 0$), a mixed equilibrium with coexistence of $P$-firms and $S$-firms dominates. The intuition here is that quantity competition with complements is akin to price competition with substitutes (e.g. Singh and Vives, 1984) and the firm which chooses a social strategy takes the role of the price leader. In consequence, this results in higher profits for both firms compared to an equilibrium with identical strategy choices (see similarly Kopel and Brand, 2012; Lambertini, 2013). If products are substitutes ($0 < \gamma < 1$), all three equilibrium outcomes can occur. Interestingly, an equilibrium where all firms are socially concerned is possible only if the degree of competition measured by degree of substitution $\gamma$ is high, but the level of social concern is not too high. In this situation where $S$ is dominant, we also have $\pi_{PP} > \pi_{SS}$ so that the game is of a prisoner’s dilemma type. In other words, even though $S$ dominates $P$, individual profits are higher when all firms are profit maximizers (see similarly Kannaiainen and Pietarila, 2006). Finally, if the degree of social concern $\theta$ is too high, it is more profitable for firms to switch to a profit-maximizing strategy. The right panel of Fig. 2 shows a situation where products are vertically differentiated ($a = 1.3$) and marginal costs are positive ($c = 0.5$). Two observations are worth pointing out. First, the region in the parameters space where $S$ dominates $P$ is now enlarged and can be divided into two sub-regions. In the light gray region the game has a prisoner’s dilemma structure as before. In the dark gray region a social strategy yields higher profits than a profit-maximizing strategy. Consequently, comparing the left and the right panels in Fig. 2, we conclude that a social strategy can increase profits if customers are willing to pay a price premium (22).

4.3. Evolutionary dynamics with Nash players

Here we focus on the evolutionary dynamics of the share of profit maximizing $P$-firms, $r(t)$, assuming that all firms are ‘Nash firms’. This means that firms select the Nash equilibrium quantity corresponding to the chosen objective function $\pi_t$ or $V_i$ in the first stage of the game (e.g. Königstein and Müller, 2001; Droste et al., 2002). After being randomly matched, each firm is able to learn the type ($P$ or $S$) of its opponent and plays the corresponding equilibrium quantity.$^6$

We assume that the share $r(t)$ follows an asynchronous updating mechanism, whose dynamics in discrete time is defined by the unidimensional map $M$ (see Hofbauer and Weibull, 1996):

$$
M: \left\{ \begin{array}{ll}
\tau(t+1) = f(\tau(t)) = \delta \tau(t) + (1 - \delta) \frac{\pi(t)e^{\theta \pi(t)}}{\pi(t)e^{\theta \pi(t)} + (1 - \tau(t))e^{\theta \pi(t)}}
\end{array} \right.
$$

$^5$ With perfectly homogeneous products ($\gamma = 1$), no vertical differentiation ($a = 1$), and zero marginal cost ($c = 0$) it is immediate to observe that condition (18) is satisfied, so that equilibrium quantities are always positive. In this case, we obtain: $S$ dominates $P$ for $\theta \in (0, \theta^*)$, Hawk-Dove game for $\theta \in (\theta^*, 3/5)$, and $P$ dominates $S$ for $\theta \in \left[\frac{3}{5}, 1\right]$, where $\theta^*$ is a solution of the equation $2\theta^3 - 9\theta^2 + 12\theta - 3 = 0$ in the interval $0, 1.$

$^6$ We weak this assumption in the next subsection by assuming that firms are not able to observe the type they are randomly matched with.
where \( \pi_P \) and \( \pi_S \) are given in (20) and (21) respectively. The parameter \( \beta \in [0, +\infty) \) is called the intensity of choice and can be interpreted as the firms’ propensity to switch strategies. The parameter \( \delta \in [0, 1] \) represents the fraction of firms per unit of time who stick to their current strategy. Thus, \( \delta = 0 \) corresponds to the case in which all firms reconsider their strategy (synchronous updating) as in Brock and Hommes (1997) and with \( \delta = 1 \) the model reduces to a non-evolutionary setting (see Hommes, 2009 for details).

The states in which all firms choose strategy \( S \) \((r_0 = 0)\) or choose strategy \( P \) \((r_1 = 1)\) are always equilibria of the unidimensional map (23). In addition, the map (23) can admit the interior equilibrium \( r^* \) in (22) where realized profits from the two strategies are equal and profit-maximizing \( P \)-firms and socially concerned \( S \)-firms coexist in the population. The next proposition shows that the asymptotic stability properties of the equilibria \( r_0 = 0 \) and \( r_1 = 1 \) are not influenced by the behavioral parameters \( \beta \) and \( \delta \). In contrast, the interior equilibrium \( r^* \) can lose stability through cascades of period-doubling bifurcations for sufficiently high values of the intensity of choice \( \beta \).

**Proposition 3.** Consider the evolutionary model described by the map \( M \) in (23).

- When strategy \( S \) dominates strategy \( P \), i.e. when \( \pi_{SS} > \pi_{PS} \) and \( \pi_{SP} > \pi_{PP} \), the map (23) admits two equilibria: \( r_0 = 0 \) (all firms are socially concerned), which is locally asymptotically stable, and \( r_1 = 1 \) (all firms are profit maximizers), which is locally asymptotically unstable.
- When strategy \( P \) dominates strategy \( S \), i.e. when \( \pi_{SS} < \pi_{PS} \) and \( \pi_{SP} < \pi_{PP} \), the map (23) admits two equilibria: \( r_0 = 0 \), which is locally asymptotically unstable, and \( r_1 = 1 \), which is locally asymptotically stable.
- When the game is Hawk-Dove (neither \( P \) dominates \( S \) nor \( S \) dominates \( P \)), i.e. when \( \pi_{SS} < \pi_{PS} \) and \( \pi_{SP} > \pi_{PP} \), the map (23) admits three equilibria: \( r_0 = 0 \) and \( r_1 = 1 \), which are both locally asymptotically unstable, and \( r^* \in (0, 1) \) given in (22), which is locally asymptotically stable for \( \beta < \beta^* \), and looses stability through a flip bifurcation at \( \beta = \beta^* \), where

\[
\beta^* = \frac{2 - \pi_{PP} - \pi_{SP} + \pi_{SS} - \pi_{PS}}{1 - \delta (\pi_{SS} - \pi_{PS})(\pi_{SP} - \pi_{PP})} > 0
\]  

(24)

- For \( \beta < \beta^* \), at \( \pi_{SS} = \pi_{PS} \) with \( \pi_{SP} > \pi_{PP} \) a transcritical (or stability exchange) bifurcation occurs, after which equilibrium \( r_0 = 0 \) becomes unstable and equilibrium \( r^* \) enters the interval \((0, 1)\) and becomes stable. Similarly, at \( \pi_{SS} = \pi_{PP} \) with \( \pi_{PS} < \pi_{SS} \) another transcritical bifurcation takes place, after which equilibrium \( r_1 = 1 \) becomes stable and \( r^* \) exits the interval \((0, 1)\).

**Proof.** See the Appendix.

This result shows that if \( S \) is a dominant strategy for the firms, the industry equilibrium where all firms are socially concerned is approached monotonically. Maybe a more realistic description of real-world industries, the proposition also demonstrates that a polymorphic state might occur where socially concerned firms and profit-maximizing firms coexist (see also Fanelli, 2010). This interior equilibrium can be approached either monotonically or non-monotonically. The model (23) even suggests that such an interior equilibrium might never be reached and the dynamics, through a cascade of period doubling bifurcations, can eventually even be chaotic. As numerical examples, consider (23) with the vertical differentiation parameter \( a = 1.05 \), marginal cost \( c = 0 \), level of social concern \( \theta = 0.3 \), and horizontal differentiation between products \( \gamma = 0.8 \). In this case we have two unstable corner equilibria \( r_0 \) and \( r_1 \). The interior equilibrium \( r^* = 0.162204 \) is independent of the ‘behavioral’ parameters \( \beta \) and \( \delta \) and the equilibrium quantities are strictly positive, see (18). According to the proposition, \( r^* \) is stable as long as \( \beta < \beta^* \), see (24). If, on average, half of the firms revise their strategy at any period of time (\( \delta = 0.5 \)), the first flip bifurcation occurs at \( \beta = \beta^*_1 = 574.579 \), after which the system moves towards chaotic behavior through the usual cascades of period-doubling bifurcations. This case is represented numerically in the bifurcation diagram in the left panel of Fig. 3, where the bifurcation parameter \( \beta \in (0, 1000) \) and initial condition \( r(0) = 0.5 \) and \( a = 1.05 \), \( c = 0 \), \( \theta = 0.3 \), \( \gamma = 0.8 \), \( \delta = 0.5 \) (right panel); \( \delta = 0 \) (left panel).

**Fig. 3.** Bifurcation diagrams for the state variable \( r \) in the evolutionary mixed duopoly with Nash players. Bifurcation parameter \( \beta \in (0, 1000) \), initial condition \( r(0) = 0.5 \) and \( a = 1.05 \), \( c = 0 \), \( \theta = 0.3 \), \( \gamma = 0.8 \), \( \delta = 0.5 \) (right panel); \( \delta = 0 \) (left panel).
(0, 1), see the bifurcation diagram in the right panel of Fig. 3. Finally, for a sufficiently high intensity of choice, the fraction \( r \) converges to the 2-cycle \( \{r_1, r_2\} = \{0, 1\} \).

4.4. Evolutionary dynamics with best reply players

In this subsection we still consider that the fraction of firms playing strategy P or strategy S evolves over time, but now we assume that firms are not able to observe the type of competitor they are randomly matched with. While we allow for different qualities of the products \( (a \geq 1) \), for the sake of simplicity we assume \( \gamma = 1 \) and marginal costs of production are zero \( (c = 0) \).

Assuming again naive expectations, i.e. \( x^P(t+1) = x_P(t) \) and \( x^S(t+1) = x_S(t) \), at time \( t \) a representative P-firm determines its quantity at time \( t+1 \) in order to maximize expected profits

\[
x^P(t+1) = \arg \max_x \mathbb{E}[\pi^P(t)] = \arg \max_x [r(t)(1-x-x_P(t))x + (1-r(t))(1-x-x_S(t))x].
\]

Analogously, for a representative S-firm we have

\[
x^S(t+1) = \arg \max_x \mathbb{E}[\pi^S(t)] = \arg \max_x \left[ \left( a - x - x_P(t) \right) x + \frac{\theta}{2} \left( x^P(t) + 2xx_P(t) + x^2 \right) \right] r + \left[ \left( a - x - x_S(t) \right) x + \frac{\theta}{2} \left( x^S(t) + 2xx_S(t) + x^2 \right) \right] (1-r). \tag{26}
\]

Under our assumptions this is equivalent to assume that each firm selects the best reply quantity corresponding to its type \((P \text{ or } S)\) to the expected population quantity in period \( t+1 \) (see also Droste et al., 2002).

\[
x^t(t+1) = r(t)x_P(t) + (1-r(t))x_S(t).
\]

The resulting equilibrium given population state \( r(t) \) is Bayesian Nash. The expected profits of choosing strategy P and strategy S at time \( t \) are, respectively, given by

\[
\begin{align*}
\hat{\pi}^P(t) &= \hat{\pi}^P(x_S(t), x_P(t), r(t)) \\
&= r(t)[(1-2x_P(t))x_P(t) + (1-r(t))(1-x_P(t)-x_S(t))x_P(t)] \\
\hat{\pi}^S(t) &= \hat{\pi}^S(x_P(t), x_S(t), r(t)) \\
&= r(t)[(1-x_P(t)-x_S(t))x_S(t)] + (1-r(t))[(1-2x_S(t))x_S(t)].
\end{align*}
\]

Coupling best-reply quantity dynamics, see (34), and asynchronous updating of firms’ strategic orientations, the dynamics is now captured by the three-dimensional map \( T \):

\[
T: \begin{cases}
x^S(t+1) = R^S(x^S(t+1)) = \frac{a - \left( r(t)x_P(t) + (1-r(t))x_S(t) \right)(1-\theta)}{2 - \theta} \\
x^P(t+1) = R^P(x^P(t+1)) = \frac{1 - \left( r(t)x_P(t) + (1-r(t))x_S(t) \right)}{2} \\
r(t+1) = \delta r(t) + (1-\delta) \frac{r(t)e^{\beta r(t)}}{r(t)e^{\beta r(t)} + (1-r(t))e^{\beta r(t)}}
\end{cases} \tag{28}
\]

It is useful to notice that if \( r(t) = 0 \) then \( r(t+1) = 0 \) for each \( t \geq 0 \). The two planes \( r=0 \) and \( r=1 \) are trapping regions, on which the dynamics is governed by the restriction of (28) to these planes. Along these manifolds, the system is either.

Note that through the linear monotone transformation

\[
y = h(x) = \frac{2 - \theta}{2(1-\theta)} x + \frac{1-a-\theta}{2(1-\theta)}
\]

it is possible to rewrite the second component of the map (28) as \( x^P(t+1) = R^P(x^S(t+1)) = h(R^P(x^S(t+1))) \). Furthermore, the equilibrium quantity for a P-firm can be expressed in terms of the equilibrium quantity of a S-firm as \( x_P = h(x_S) \).\(^8\)

The map (28) always has two corner equilibria:

1. \( E_0 = (x^P_0, x^S_0, 0) = (a/(3-2\theta), (3 - a - 2\theta)/(6 - 4\theta), 0) \). This corner equilibrium corresponds to the equilibrium \((x_{SS}, x_{SS})\) where every firm plays strategy S; see also (16).
2. \( E_1 = (x^P_1, x^S_1, 1) = ((3a+\theta-1)/(3(2-\theta)), 1/3, 1) \). This corner equilibrium corresponds to the equilibrium \((x_{PP}, x_{PP})\) where every firm plays strategy P; see also (15).

\(^{8}\) See the best replies given in (34) in the Appendix.

\(^{8}\) This is straightforward, since in equilibrium we have \( x_S = R^S(r^P x_P + (1-r^P) x_S) \) and \( x_P = R^P(r^S x_P + (1-r^S) x_S) \) \( = h(R^S(r^S x_P + (1-r^S) x_S)) = h(x_S) \).
Depending on the parameters of the model, the map (28) can also admit an interior equilibrium $E^* = (x_S^*, x_P^*, r^*)$, with coexistence of P-firms and S-firms:

$$x_S^* = \frac{a(\theta(2\theta - 3) + 2) - (\theta - 3)\theta - 2 - 2\sqrt{\Psi}}{\theta^2}$$

$$r^* = 2 - \frac{a}{a + \theta - 1} \frac{3(3a + 2\theta - 3)\sqrt{\Psi}}{(a - 1)(\theta - 1)\theta(a + \theta - 1)}$$

(30)

where $x_S^* = h(x_S^*)$, see (29), and $\Psi = (a - 1)(\theta - 1)^2(a(\theta - 1)\theta + 1) + (a - 1)(\theta - 1)$ (30). Notice that there is a difference between the equilibrium (22) with Nash players and the equilibrium (30) with best-reply players. The reason is that in the interior equilibrium with Nash players (22) firms still play the pure strategy Cournot quantities in (17), whereas in the model with best reply players firms choose the best response to the expected output in the market. The equilibrium $E^*$ is economically feasible provided that $r^* \in (0, 1)$ and $x_S^* > 0$ and $x_P^* = h(x_S^*) > 0$. That is, P-firms and S-firms coexist in the market and both type of firms produce positive quantities. Notice from (30) that if $a = 1$, the interior equilibrium (30) is undefined. If $a > 1$ the equilibrium is meaningful provided that $\theta \in (\theta_1, \theta_2)$, where

$$\theta_1 = \frac{3 - a}{2} - a^2 + a\sqrt{a^2 + a - 2}$$

$$\theta_2 = \frac{1}{2} \left( \sqrt{9a^2 - 6a - 3} - 3a + 3 \right)$$

(31)

Having discussed conditions of existence for equilibria, we now study the stability properties. We first focus on the corner equilibria $E_0$ and $E_1$. As in the case of Nash players, a boundary equilibrium where all firms select the same strategic goal, be it S or P, is locally asymptotically stable if and only if the payoff obtained by pursuing that particular goal is greater than the payoff a firm would obtain by pursuing the alternative goal (with Cournot equilibrium quantities). Stated differently, stability of a corner equilibrium occurs whenever there is no incentive for a single firm to deviate from the current state. Thus, stability and profitability are equivalent for corner equilibria. Moreover, the stability properties of these equilibria are not influenced by the parameters of the evolutionary dynamics $\delta$ and $\beta$.

**Proposition 4.** Consider the evolutionary model described by the map $T$ in (28) and $\theta_1$ and $\theta_2$ given in (31).

- **When** $a > 1$ and $0 < \theta < \theta_1$, the model admits two equilibria: $E_0 = (x_S^0, x_P^0, 0)$, which is locally asymptotically stable and $E_1 = (x_S^1, x_P^1, 1)$, which is a saddle point.
- **When** $a > 1$ and $\theta_1 < \theta < \theta_2$, the model admits three equilibria: $E_0$, $E_1$, and the inner equilibrium $E^*$, where $E_0$ and $E_1$ are both saddle points.
- **When** $a > 1$ and $\theta_2 < \theta \leq 1$, or when $a = 1$ and $0 < \theta \leq 1$, the model admits two equilibria: $E_1$, which is locally asymptotically stable and $E_0$, which is a saddle point.

**Proof.** See the Appendix.

The results in the proposition provide the following economic insights. If there is no vertical (and, by assumption, no horizontal) differentiation, $a = 1$, we have $\theta_1 = \theta_2 = 0$. Consequently, for any level of social concern, $0 < \theta \leq 1$, the equilibrium $E_1 = (x_S^1, h(x_S^1), 1)$ is always locally asymptotically stable. In such a situation it is not profitable for firms to be socially concerned since consumers are not willing to pay a price premium for products offered by socially concerned firms. In equilibrium, the industry only consists of profit-maximizing firms. In contrast, if $a > 1$, we obtain $\theta_1 < \theta_2$. If the level of social concern $\theta$ is not too high ($0 < \theta < \theta_1$), then the equilibrium $E_0 = (x_S^0, h(x_S^0), 0)$ in which all firms are socially concerned is locally asymptotically stable. In line with Gary Becker’s comment reported in the introduction, if consumers are willing to pay more for the products of S-firms it pays off to be socially concerned. According to our result, the equilibrium $E_1$ is a saddle point. It is stable only with respect to the ‘quantity’ dynamics but not with regard to the ‘evolutionary’ dynamics. In other words, if all firms act as profit maximizers, then in the long run the industry would consist of P-firms choosing the Cournot equilibrium quantity (as a set of the form $(x_S(t), x_P(t), 1)$ is invariant for (28)). On the other hand, if there are some firms which are socially concerned, this may lead profit-maximizing firms to switch and become socially concerned. This is due to the fact that $E_0$ is locally stable if strategy S outperforms the alternative strategy P in a neighborhood of the equilibrium. In effect, this may lead to a “herding effect” where more and more firms are adopting a social strategy. As a final insight from our results, if the level of social concern $\theta$ is too high (i.e. $\theta_2 < \theta \leq 1$), then the equilibrium $E_1$ with all profit maximizers is locally asymptotically stable. Therefore, social concern is rewarded only up to a point or as Mintzberg (1983) puts it: “it pays to be good, but not too good” (p. 7). The equilibrium $E_0$ with all socially concerned firms is a saddle point (analogous to the previous case), because strategy P dominates strategy S in a neighborhood of the equilibrium. Note that, also in this case, the parameters $\delta$ and $\beta$ play no role whatsoever for the stability of the two corner equilibria.

In contrast to the corner equilibria, the interior equilibrium $E^*$, where profit maximizers and socially concerned firms coexist, has different stability properties. Since the Jacobian matrix at this equilibrium and the stability conditions are quite involved, we rely on numerical and graphical arguments to study the dynamics of the map (28). Fig. 4 depicts the $(x_S, r)$--plane and shows the graphs of two equations: (1) $R^D(rh(x_S) + (1 - r)x_S) = x_S$, see (29), and (2) $\tilde{R}_P = \tilde{R}_S$, with $x_P = h(x_S)$,
see (27). These two graphs intersect at $E^*$ and are useful to shed some light on the stability properties of $E^*$ (the ‘evolutionary’ parameters $\beta$ and $\delta$ do not affect the location of these two curves). Consider the graph of the first curve. Starting from a point $(x_S(t), x_P(t), r(t))$, with $x_P(t) = h(x_S(t))$, an iteration of the map (28) generates a new point $(x_S(t+1), x_P(t+1), r(t+1))$. To the left of the first curve, we have $x_S(t+1) = R^S(r(t)h(x_S(t)) + (1 - r(t))x_S(t)) > x_S(t)$. Hence, the quantity of a socially concerned firm increases. To the right of this curve, the opposite statement holds. Similarly, the second curve represents the curve where the two strategies $P$ and $S$ yield identical profits. To the right of the second curve, we have $x_P(t) > x_S(t)$, which implies that $r(t+1) > r(t)$ because of the replicator equation in (28). Again, the opposite statement holds to the left of the second curve. Obviously, the intersection point of the two curves gives the unique interior equilibrium $E^*$ in (30). Summarizing, the previous arguments allow us to obtain qualitative insights on the dynamic behavior of the system. As shown by the arrows in Fig. 4, where $a = 1.8$ and $\theta = 0.615$, $E^*$ is a candidate to be a stable equilibrium. This is a situation where the intensity of choice $\beta$ and/or the fraction of switching agents $1 - \delta$ is sufficiently low. From the conditions for the existence of $E^*$ and the stability conditions of Proposition 3 for the corner equilibria, it follows that $E^*$ is created through a transcritical (or stability exchange) bifurcation at $\theta_1$, where $E_0$ and $E^*$ collide and interchange their stability properties. After this, $E^*$ is the only possible stable steady state of the system. Similarly, at $\theta_2$, another transcritical bifurcation occurs at which $E^*$ and $E_1$ merge, with $E_1$ the only stable equilibrium of the system.

Similar to the case of Nash players, the interior equilibrium $E^*$ can lose stability for high intensity of choice $\beta$ and/or a high share of agents revising their strategy over time, i.e. a low $\delta$. Consider a typical example using parameter values $a = 1.05$ and $\theta = 0.3$. In this case the interior equilibrium is $E^* = (0.4594, 0.3078, 0.4953)$. Although the location of $E^*$ does not depend on $\delta$ and $\beta$, these parameters do influence its stability. The upper part of Fig. 5 shows a bifurcation diagram with bifurcation parameter $\beta \in (0, 1500)$, initial condition $(x_S(0), x_P(0), r(0)) = (0.5, 0.3, 0.7)$ and $\delta = 0.01$ (i.e., on average 99% of the firms revise their strategy at each time period). The interior equilibrium $E^*$ loses stability at $\beta_1 \approx 116.6$ through a flip bifurcation, which represents a typical case of overshooting (or overreaction) in the evolutionary switching process. Numerical evidence shows that the resulting 2-cycle does not lose stability as $\beta$ is further increased. Notice that, similar to the model (23) with Nash players, the 2-cycle in the state variable $r$ converges to $[r_1, r_2] = (0, 1)$ for sufficiently high intensity of choice (see the top-right panel in Fig. 5). The lower panel of Fig. 5 is based on the same parameter set, but now every firm, on average, revises its strategy ($\delta = 0$). In this case, and differently from the model with Nash players, the 2-cycle can lose stability through a Neimark–Sacker bifurcation. As the intensity of choice $\beta$ is increased, cyclic and chaotic dynamics can be observed. We conclude by remarking that this kind of loss of stability of the 2-cycle has been detected in most of the numerical experiments performed with synchronous switching ($\delta = 0$).

5. Conclusions

In this paper we have studied the endogenous selection of the objective of business firms if shareholder value maximization is key. Using an evolutionary model with asynchronous switching between a social and a profit-maximizing motive we have shown that it can pay off for shareholders to implement a social strategy emphasizing consumer welfare if customers are willing to pay a price premium for the socially concerned firm’s product. Depending on the degree of product
differentiation and the weight put on consumer welfare, we have observed industry configurations comprising profit-maximizing and socially concerned firms. This finding also contributes an explanation for the empirically documented heterogeneity of firms within industries and markets. We have also found other situations where just one type of firm dominates the industry in the long run and identified the conditions for this to occur.

Only few papers in industrial economics are using dynamic, evolutionary models. This is in stark contrast to the assessment of Cabral (2012), who argues that, "...dynamic oligopoly models are an area where much work needs to be done and much work can be done.," and further, "...dynamic oligopoly models provide considerable value added with respect to static models." (p. 278). In this vein, our paper contributes to the field by analyzing a dynamic mixed oligopoly model. Due to space constraints many important questions had to be left for future research. First, in this paper we have focussed exclusively on local stability and local bifurcations. It would however be interesting to study the global dynamics (e.g. Bischi et al., 2010; Brito et al., 2012; Droste et al., 2002). This would enable us to understand the impact of the initial industry configuration on the evolutionary long run outcome and the basin boundaries of the resulting attracting sets. Second, in line with the evolutionary literature (e.g. Bischi et al., 2003a,b; Droste et al., 2002) we have restricted ourselves to two-firm matching, i.e. in essence to a mixed duopoly market. Again, it would be interesting to extend our analysis along the lines of Hommes et al. (2011) and study a more general N-firm setting. The challenge is that the resulting quantity best reply dynamics is not converging to the Nash equilibrium if the number of firms is larger than two and may potentially be solved by resorting to a different quantity adjustment process. Finally, our model just considers two types of organizational structures. In line with models on evolutionary switching between several (more or less) costly behavioral (or expectation) rules, it would be interesting to study the relative fitness of several (costly) hybrid organizational structures like (partially privatized) public firms or cooperatives under evolutionary competition. This would be a legitimate test bed for results obtained for the variety of static models on mixed oligopoly markets.

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Appendix A

A.1. Proof of Proposition 2

We denote by \( R^{XY} \) the reaction function of an \( X \)-firm when its opponent is a \( Y \)-firm, with \( X, Y \in \{S, P\} \). The corresponding expressions for the reaction functions follow from (7) and (9).
The case where both firms are of P-type corresponds to the classic Cournot model, in which
\[
x_{pj}(t+1) = R^{pp}(x_{pj}(t)) = \frac{1 - c - x_{pj}(t)}{2},
\]
which is a contraction implying stability for \(N=2\).

In the case where both firms select to be an S-type, the best reply dynamics is given by the bidimensional linear map
\[
x_{sj}(t+1) = R^{ss}(x_{sj}(t)) = \frac{a - c - x_{sj}(t)(1 - \theta)}{2 - \theta}
\]
for \(i, j = 1, 2, i \neq j\), (33)
which is a contraction, since \((1-\theta)/(2 - \theta) < 1\) for all \(\theta \in [0, 1]\).

In the mixed case where a P-firm plays against a S-firm, the best reply dynamics reads
\[
\begin{aligned}
x_{s1}(t+1) &= R^{sp}(x_{p1}(t)) = \frac{a - c - \gamma x_{p1}(t)(1 - \theta)}{2 - \theta} \\
x_{p1}(t+1) &= R^{ps}(x_{s1}(t)) = \frac{1 - c - \gamma x_{s1}(t)}{2}
\end{aligned}
\]
(34)
where \(x_{s1}(t)\) and \(x_{p1}(t)\) are the quantities produced by the S-firm and the P-firm respectively at time \(t\). This is again a contraction, since \(\gamma(1-\theta)/(2 - \theta)\) and \(\gamma/2\) are in \((-1, 1)\) for all \(\theta \in [0, 1]\) and \(\gamma \in [-1, 1]\).

### A.2. Proof of Proposition 3

Consider, the case in which strategy \(S\) dominates strategy \(P\). For the map \((28)\), it is immediate to observe that
\[
f'(r_0) = \delta + (1 - \delta)e^{-\beta S-S-S-P} \in (0, 1)
\]
(35)
being \(\pi_{SS} > \pi_{PS}\), whereas
\[
f'(r_1) = \delta + (1 - \delta)e^{-\beta S-P-S-P} \in (1, +\infty)
\]
(36)
being \(\pi_{SP} > \pi_{PP}\). Hence, \(r_0 = 0\) is locally stable and \(r_1 = 1\) is locally unstable. The same reasoning applies when strategy \(P\) dominates strategy \(S\) to show that \(r_0\) is an unstable fixed point and \(r_1\) is a stable one.

Now consider the case in which the game is Hawk-Dove \((\pi_{PS} > \pi_{SS} \text{ and } \pi_{SP} > \pi_{PP})\). From these inequalities, it follows that equilibrium \(r^*\) in \((22)\) is well-defined, that is, \(r^* \in (0, 1)\). From \((35)\) and \((36)\), \(r_0 \text{ and } r_1\) are both unstable fixed points. Moreover, at \(r^*\) we have
\[
f'(r^*) = 1 - \beta(1-\delta)\frac{[\pi_{SS} - \pi_{PS}][\pi_{SP} - \pi_{PP}]}{\pi_{PP} - \pi_{SP} + \pi_{SS} - \pi_{PS}} < 1
\]
where the last inequality follows from the fact that the expression in square brackets is strictly positive when \(r^*\) belongs to the interval \((0, 1)\). Thus, the interior equilibrium \(r^*\) is locally asymptotically stable (unstable) for \(\beta > \beta^*\) (\(\beta < \beta^*\)), and looses stability through a flip bifurcation at \(\beta = \beta^*\). Since with asynchronous switching the intensity of choice \(\beta \in (0, +\infty)\), the interior equilibrium always loses stability for a sufficiently high level of \(\beta\) through period doubling. Notice that \(r^*\) is created and destroyed through two transcritical bifurcations: at \(\pi_{SS} = \pi_{PS}\) with \(\pi_{SP} > \pi_{PP}\), the equilibrium \(r_0\) becomes unstable and \(r^*\) becomes positive and stable if \(\beta < \beta^*\). Similarly, at \(\pi_{SP} = \pi_{PP}\) with \(\pi_{SS} < \pi_{PS}\), \(r_1\) becomes stable and \(r^*\) leaves the interval \((0, 1)\).

### A.3. Proof of Proposition 4

At \(E_0\), the Jacobian of \((28)\) reads
\[
J(E_0) = \begin{pmatrix}
\frac{1 - \theta}{2 - \theta} & 0 & \frac{(1-\theta)x_3 - 2\theta}{2(2-\theta)x_3 - 2\theta} \\
-1 & 0 & \frac{\theta}{2(2-\theta)x_3 - 2\theta} \\
0 & 0 & \delta + (1 - \delta)e^{\theta(3 - \theta^2)}
\end{pmatrix}
\]
so that its eigenvalues at \(E_0\) are the entries in the main diagonal, \(J_{11}(E_0) = (1 - \theta)/(2 - \theta) \in (-1, 0)\), \(J_{22}(E_0) = 0\) and \(J_{33}(E_0) \in (0, +\infty)\). Thus, local stability of \(E_0\) occurs for \(J_{22}(E_0) \in (0, 1) \iff \tilde{r}_p(x_3^0, h(x_3^0), 0) - \tilde{r}_S(x_3^0, h(x_3^0), 0) = ((3 - \theta^2) + a(4\theta - 6) + a^2(8\theta - 3)/4(3 - \theta^2) < 0\), that is for \(\theta \in (0, \theta_1)\), where \(\theta_1\) is given in \((31)\). The analysis at \(E_1\) is similar: the Jacobian of \((28)\) at \(E_1\) is
\[
J(E_1) = \begin{pmatrix}
0 & \frac{1 - \theta}{2 - \theta} & \frac{(1-\theta)x_3 + 2\theta}{3(2 - \theta)} \\
-1 & 0 & \frac{2\theta + 2\theta^2}{6(2 - \theta)} \\
0 & 0 & \delta + (1 - \delta)e^{\theta(3 - \theta^2)}(3\theta(1 - \theta) + (3 - \theta^2)(9\theta - 2)^2 < 0\) or \(\theta \in (\theta_2, 1)\), where again \(\theta_2\) is given in \((31)\).